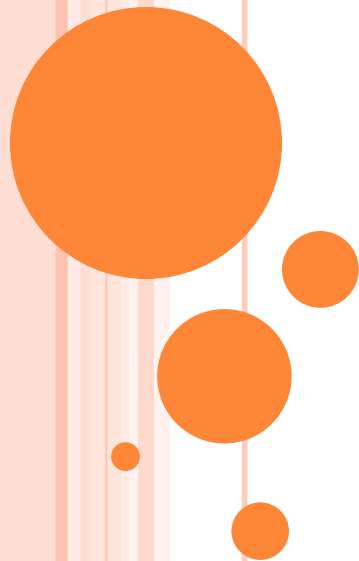


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Theory of Matrix

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Government of India
Ministry of Human Resource Development
Department of Higher Education

Pay Matrix Proposed for IITs/IISc/IIM/NITIE/IISER/NIT/IIIT – In 4-tier structure* (All figures are in Rupees)

Cadre Title		Asst. Prof. Grade II		Asst. Prof. Grade I		Associate Prof.	Professor		
6 th PC Pay Band		PB3 15600-39100				PB4 37400-67000			67000-79000
Grade Pay – IIT etc.		6000	7000	8000	9000**	9500	10000	10500	HAG
Index of Rationalisation		2.67	2.67	2.67	2.67	2.67	2.72	2.72	2.72
Entry Pay IIT etc.		21600	25790	38000	49200	52300	53000	58500	67000
Cell No.	Pay Level	10	11	12	13A1	13A2	14	14A	15
1		57700	68900	101500	131400	139600	144200	159100	182200
2		59400	71000	104500	135300	143800	148500	163900	187700
3		61200	73100	107600	139400	148100	153000	168800	193300
4		63000	75300	110800	143600	152500	157600	173900	199100
5		64900	77600	114100	147900	157100	162300	179100	205100
6		66800	79900	117500	152300	161800	167200	184500	211300
7		68800	82300	121000	156900	166700	172200	190000	217600
8		70900	84800	124600	161600	171700	177400	195700	224100
9		73000	87300	128300	166400	176900	182700	201600	
10		75200	89900	132100	171400	182200	188200	207600	
11		77500	92600	136100	176500	187700	193800	213800	
12		79800	95400	140200	181800	193300	199600	220200	
13		82200	98300	144400	187300	199100	205600		
14		84700	101200	148700	192900	205100	211800		
15		87200	104200	153200	198700	211300			
16		89800	107300	157800	204700				
17		92500	110500	162500					
18		95300	113800	167400					
19		98200	117200						

*As ISM, Dhanbad has become IIT, not shown separately; ** 9000 grade pay also has Asso. Prof (pre 4-tier), not shown separately.



INTRODUCTION

Matrix

Definition:

- ❖ Matrix is the structural organization of numbers.
- ❖ it is a rectangular array of numbers which is arranged in a rows and columns.
- ❖

It is denoted by () or [] symbols.

The array of numbers below is an example of matrix.

a_{11}	a_{12}	a_{13}	$a_{14}.....$	a_{1n}
a_{21}	a_{22}	a_{23}	$a_{24}.....$	a_{2n}
a_{31}	a_{32}	a_{33}	$a_{34}.....$	a_{3n}
.....
.....
a_{m1}	a_{m2}	a_{m3}	$a_{m4}.....$	a_{mn}



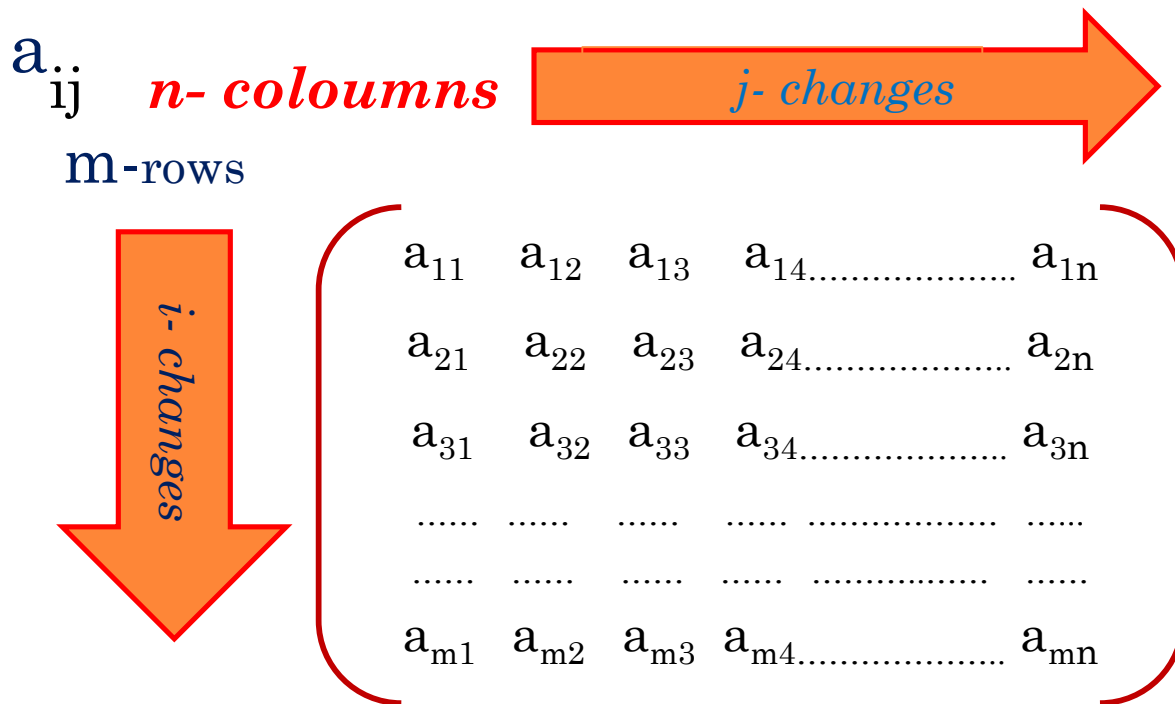
In This matrix

- ❖ There are **m-rows** and **n-coloums**.
- ❖ The **order** (dimension) of Matrix is **m x n**
- ❖ The **numbers** a_{11} a_{12} a_{13} a_{ij} ... are called elements (or entries) of the matrix.
- ❖ Element **a_{ij}** refers to the element located in the **i**-th row and **j**-th coloumn
- ❖ Thus the above matrix is expressed as

$$A=[a_{ij}]_{m \times n}$$

Where $i= 1,2,3,\dots,m$
 $j=1,2,3,\dots,n$

m-by-n Matrix



An English mathematician named Cullis was the first to use modern bracket notation for matrices in 1913

and

he simultaneously demonstrated the first significant use of the notation $\mathbf{A} = [a_{ij}]$ to represent a matrix where a_{ij} refers to the i th row and the j th column

The term "matrix" (Latin for "womb", derived from mater—mother) was coined by James Joseph Sylvester in 1850, who understood a matrix as an object giving rise to a number of determinants today called minors, that is to say, determinants of smaller matrices that derive from the original one by removing columns and rows. In an 1851 paper, Sylvester explains: I have in previous papers defined-

a "Matrix" as a rectangular array of terms, out of which different systems of determinants may be engendered as from the womb of a common parent.

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 9 & 10 \end{pmatrix}$$

➤ The number of rows and columns that a matrix has is called its **dimension** or its **order**.

➤ By convention, rows are listed first; and columns, second.

Thus, we would say that the dimension (or order) of the above matrix is 3 X 3, meaning that it has 3 rows and 3 columns.

➤ # Numbers that appear in the rows and columns of a matrix are called **elements** of the matrix.

➤ In the above matrix, the element in the first column of the first row is 1; the element in the second column of the first row is 2; and so on.

Matrix notation

We use symbols to identify matrix elements and matrices.

Matrix elements: Consider the matrix below, in which matrix elements are represented entirely by symbols.

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

By convention, first subscript refers to the **row number**; and the second subscript, to the **column number**.

Thus, the first element in the first row is represented by a_{11} . The second element in the first row is represented by a_{12} . And so on, until we reach the fourth element in the second row, which is represented by a_{22} .

Matrices There are several ways to represent a matrix symbolically. The simplest is to use a boldface letter, such as **A**, **B**, or **C**. Thus, **A** might represent a 3X3 matrix, as illustrated below.

$$\mathbf{A} = \begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{4} & \mathbf{5} & \mathbf{6} \\ \mathbf{8} & \mathbf{9} & \mathbf{10} \end{pmatrix}$$



Another approach for representing matrix **A** is:

$$\mathbf{A} = [a_{ij}],$$

Where $i = 1, 2, 3$ and $j = 1, 2, 3$

This notation indicates that **A** is a matrix with 3 rows and 3 columns. The actual elements of the array are not displayed; they are represented by the symbol a_{ij} .

Matrix Equality:

To understand matrix algebra, we need to understand matrix equality. Two matrices are equal if all three of the following conditions are met.

- # Each matrix has the same number of rows.
- # Each matrix has the same number of columns.
- # Corresponding elements within each matrix are equal.



2. Type of Matrices:-

- (a) **Square matrix**
- (b) **Non- square matrix**
- (c) **Row matrix**
- (d) **Column matrix**
- (e) **Triangular matrix:-** Triangular matrix are four types
 - (i) **Upper triangular matrix**
 - (ii) **Lower triangular matrix**
 - (iii) **Strictly upper triangular matrix**
 - (iv) **Strictly lower triangular matrix**



(f) **Symmetric matrix**

(g) **Skew-symmetric matrix**

Remark: Every square matrix can be expressed as sum of symmetric and skew-symmetric matrix.

(h) **Diagonal matrix**

(i) **Identity matrix**

(j) **Scalar matrix**

Remark: -

Every scalar matrix is also a diagonal matrix but if the diagonal elements are 1 of the scalar matrix then it is also an identity matrix. In other words, we can say that every identity matrix is a diagonal and scalar matrix.



TYPES OF MATRICES

1. Row matrix or vector

Any number of columns but only one row

$$\begin{bmatrix} 1 & 1 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3 & 5 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \cdots a_{1n} \end{bmatrix}$$



TYPES OF MATRICES

2.Column matrix or vector:

The number of rows may be any integer but the number of columns is always 1

$$\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$$



TYPES OF MATRICES

3. Rectangular matrix

Contains more than one element and number of rows is not equal to the number of columns

$$\begin{bmatrix} 1 & 1 \\ 3 & 7 \\ 7 & -7 \\ 7 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 3 & 3 & 0 \end{bmatrix}$$

$$m \neq n$$



TYPES OF MATRICES

4. Square matrix

The number of rows is equal to the number of columns

(a square matrix \mathbf{A} has an order of m)

$$\begin{matrix} m \times m \\ \begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix} \end{matrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 9 & 9 & 0 \\ 6 & 6 & 1 \end{bmatrix}$$

The principal or main diagonal of a square matrix is composed of all elements a_{ij} for which $i=j$



TYPES OF MATRICES

5. Diagonal matrix

A square matrix where all the elements are zero except those on the main diagonal

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i \neq j$ /

$a_{ij} \neq 0$ for some or all $i = j$

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$



TYPES OF MATRICES

6. Unit or Identity matrix - I

A diagonal matrix with ones on the main diagonal

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a_{ij} & 0 \\ 0 & a_{ij} \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i \neq j$ /

$a_{ij} = 1$ for some or all $i = j$



TYPES OF MATRICES

7. Null (zero) matrix - 0

All elements in the matrix are zero

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$a_{ij} = 0 \quad \text{For all } i,j$$



TYPES OF MATRICES

8. Triangular matrix

A square matrix whose elements above or below the main diagonal are all zero

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 8 & 9 \\ 0 & 1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$



TYPES OF MATRICES

8a. Upper triangular matrix

A square matrix whose elements below the main diagonal are all zero

$$\begin{bmatrix} a_{ij} & a_{ij} & a_{ij} \\ 0 & a_{ij} & a_{ij} \\ 0 & 0 & a_{ij} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 8 & 7 \\ 0 & 1 & 8 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 7 & 4 & 4 \\ 0 & 1 & 7 & 4 \\ 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i > j$



TYPES OF MATRICES

8b. Lower triangular matrix

A square matrix whose elements above the main diagonal are all zero

$$\begin{bmatrix} a_{ij} & 0 & 0 \\ a_{ij} & a_{ij} & 0 \\ a_{ij} & a_{ij} & a_{ij} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i < j$



TYPES OF MATRICES

9. Scalar matrix

A diagonal matrix whose main diagonal elements are equal to the same scalar

A scalar is defined as a single number or constant

$$\begin{bmatrix} a_{ij} & 0 & 0 \\ 0 & a_{ij} & 0 \\ 0 & 0 & a_{ij} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

i.e. $a_{ij} = 0$ for all $i \neq j$ ✓
 $a_{ij} = a$ for all $i = j$



3. MATRIX OPERATIONS

- (a) Matrix addition**
- (b) Matrix subtraction**
- (c) Matrix multiplication**
- (d) Elementary operation**
 - (i) Elementary row operation**
 - (ii) Elementary column operation**



4. MATRIX PROPERTIES

(i) **Vector dependence:**

(ii) **Matrix rank:** No. of non- zero rows in echelon form is called rank of the matrix.

(iii) **Matrix determinant:** Matrix determinant is a value (i.e. scalar quantity).

(iv) **Matrix inverse:**



5. CONCLUSION: USE AND ITS APPLICATIONS

- (a) Matrix is used in **advanced statistics**, largely because it provides two benefits.
 - (i) Compact notation for describing sets of data and sets of equations.
 - (ii) Efficient methods for manipulating sets of data and solving sets of equations.
- (b) **Operation Research (In LPP : Transportation problems, Assignment problems, game theory etc. In NLPP Hessian Matrix used function is convex/concave)**
- (c) **Networking Theory**
- (d) **System Theory , Control Theory**
- (e) **Summation, Mean scores, Deviation scores, Variance-Covariance etc.**



Distance Matrix of Cities of California (U.S.)

Eureka	Los Angeles	Palm Springs	S Lake Tahoe	Sacramento	San Diego	San Francisco	Santa Barbara	Yosemite NP	
	659	788	393	304	776	278	599	476	Eureka
	1061	1268	632	489	1249	447	964	766	Los Angeles
659		103	441	383	127	403	96	307	Palm Springs
1061		166	710	616	204	649	154	494	S Lake Tahoe
788	103		474	484	135	504	199	408	Sacramento
1268	166		763	779	217	811	320	657	San Diego
353	467	510		135	554	222	536	213	San Francisco
568	752	821		217	892	357	863	343	Santa Barbara
304	383	484	99		510	87	391	172	Yosemite NP
489	616	779	159		821	140	629	277	
776	127	135	529	510		548	228	434	
1249	204	217	851	821		882	367	698	
278	403	504	182	87	548		321	184	
447	649	811	293	140	882		517	296	
599	96	199	508	391	228	321		329	
964	154	320	817	629	367	517		529	
476	307	408	159	172	434	184	329		
766	494	657	256	277	698	296	529		

Miles
Kilometers

Rajouri Garden	15	10	17	12	18	13	21	21	12	19	21	19	14	16	4	11	9	27	13	4	13	20	22	22	4	11	0	12	13	24	16	17	13	5
R K Puram Sector-12	5	17	9	10	8	12	7	15	7	8	10	12	15	10	13	14	10	21	18	7	11	10	13	16	13	0	11	24	9	14	5	7	3	14
Punjabi Bagh	18	7	17	12	19	12	18	11	14	21	23	18	13	15	7	10	8	26	8	6	12	22	12	21	0	13	4	10	22	24	18	20	15	7
Patparganj	12	18	9	9	9	10	10	14	22	11	13	27	12	6	24	14	12	6	17	20	9	11	10	0	21	16	21	26	17	12	11	21	18	25
Old Delhi Railway Stn.	13	8	10	4	11	1	11	5	68	14	16	23	3	5	16	5	5	16	8	13	2	14	0	10	12	13	13	16	18	16	12	20	15	17
Nehru Place	6	22	6	12	3	14	4	18	16	2	2	21	16	10	22	18	16	16	22	16	13	0	14	11	22	9	20	29	6	6	4	13	10	23
New delhi Railway Stn.	11	9	8	2	10	1	10	6	16	12	15	21	4	4	16	6	4	15	9	12	0	13	2	9	12	11	13	17	16	15	11	18	13	17
Naraina	13	11	13	10	14	13	13	12	8	15	17	15	14	14	8	11	8	25	13	0	12	16	13	20	6	7	4	14	16	20	12	13	9	9
Model Town	19	4	17	10	18	8	18	4	21	21	23	27	8	12	16	4	9	23	0	13	9	22	8	17	8	18	12	11	24	24	19	24	20	16
Mayur Vihar Phase-III	18	24	15	15	14	15	15	20	28	16	18	33	17	11	30	20	18	0	23	26	15	16	16	6	26	21	27	31	22	14	16	26	23	31
Karol Bagh	12	7	10	4	12	5	12	6	14	15	17	20	7	7	12	6	0	19	9	8	4	15	5	13	8	10	9	14	17	18	12	17	12	13
Kamla Nagar	16	5	13	7	15	5	15	1	19	18	20	24	5	9	14	0	6	20	4	11	6	18	4	15	10	14	11	13	20	20	15	21	17	15
Janakpuri Dist. Centre	17	13	19	16	20	17	19	16	12	21	22	17	18	19	0	15	13	30	16	8	17	22	16	25	7	13	5	21	21	26	18	18	14	2
I.T.O	10	12	5	4	7	4	7	9	18	9	12	22	6	0	19	9	7	12	12	14	4	10	5	6	15	10	15	20	14	12	8	17	13	20
I.S.B.T (Kashmere Gate)	15	9	11	6	12	3	13	4	19	15	17	24	0	6	18	5	7	17	8	14	4	16	3	12	13	15	14	17	19	18	14	22	17	19
I.G.I Airport	17	25	20	19	19	23	19	24	8	20	20	0	25	21	17	24	20	33	27	14	21	21	23	27	21	12	18	29	16	26	17	8	12	19
Greater Kailash-II	6	23	8	14	5	16	6	20	15	3	0	20	17	12	22	20	17	18	23	17	15	2	16	31	23	10	21	13	6	7	6	12	9	24
Greater Kailash-I	4	21	5	11	3	13	3	18	15	0	3	20	15	10	21	18	15	18	21	15	21	2	14	12	20	8	19	28	6	8	4	12	9	22
Domestic Airport	11	18	14	14	14	17	13	18	0	14	15	8	19	19	12	18	14	27	21	8	15	15	17	21	14	6	12	22	13	20	11	6	6	13
Delhi University	16	5	14	11	15	5	15	0	19	18	20	24	5	9	16	1	6	20	4	12	6	19	4	14	11	15	12	13	21	21	16	22	17	16
Defence Colony	3	18	2	8	1	11	0	15	13	3	6	19	12	7	19	15	12	15	18	13	10	4	11	9	17	7	17	25	7	8	2	12	8	20
Chandni Chowk	12	8	9	4	11	0	11	5	17	13	16	23	3	4	17	5	5	15	8	13	1	14	1	10	13	12	13	16	17	16	12	19	15	18
Central Mrkt. (Lajpat Ngr.)	4	19	3	10	0	11	1	15	14	3	5	20	12	7	20	15	12	14	18	14	10	3	11	9	19	8	18	26	8	7	3	12	9	12

Mumbai Local Train Fare Matrix

Fare Chart (Rs)												
	To											
From	VER	DNG	AZN	AND	WEH	CHK	APR	MAN	SAN	ASA	JNG	GHA
VER	10	10	20	30	40	50	60	70	80	90	100	110
DNG	10	10	10	20	30	40	50	60	70	80	90	100
AZN	20	10	10	10	20	30	40	50	60	70	80	90
AND	30	20	10	10	10	20	30	40	50	60	70	80
WEH	40	30	20	10	10	10	20	30	40	50	60	70
CHK	50	40	30	20	10	10	10	20	30	40	50	60
APR	60	50	40	30	20	10	10	10	20	30	40	50
MAN	70	60	50	40	30	20	10	10	10	20	30	40
SAN	80	70	60	50	40	30	20	10	10	10	20	30
ASA	90	80	70	60	50	40	30	20	10	10	10	20
JNG	100	90	80	70	60	50	40	30	20	20	10	10
GHA	110	100	90	80	70	60	50	40	30	30	20	10

Use of matrices in Railways (Fare Display)

Fare Structure for Rajdhani and Duronto category of Trains										
Charges % of berths	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
2S	1X	1.1X	1.2X	1.3X	1.4X	1.5X	1.5X	1.5X	1.5X	1.5X
SL	1X	1.1X	1.2X	1.3X	1.4X	1.5X	1.5X	1.5X	1.5X	1.5X
3A	1X	1.1X	1.2X	1.3X	1.4X	1.4X	1.4X	1.4X	1.4X	1.4X
2A	1X	1.1X	1.2X	1.3X	1.4X	1.5X	1.5X	1.5X	1.5X	1.5X
1A	1X	1X	1X	1X	1X	1X	1X	1X	1X	1X
X= Base Fare										
Fare Structure for Shatabdi category of Trains										
Charges % of berths	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
CC	1X	1.1X	1.2X	1.3X	1.4X	1.5X	1.5X	1.5X	1.5X	1.5X
EC	1X	1X	1X	1X	1X	1X	1X	1X	1X	1X
X= Base Fare										

