Complex Analysis

Dr. Brij Mohan Yadav
Assistant Professor
(Deptt. Of Mathematics)

Transformation or Mapping

- The equations-

$$\left.\begin{aligned} u &= u(x,y) \\ v &= v(x,y) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(1)$$

define a *transformation* or *mapping* by means of which a correspondence between *uv*-plane and *xy*-plane can be established. If corresponding to each point of *xy*- plane, there is a unique point of uv-plane and conversely, then such *transformation* is called *one-one transformation*. The corresponding points in the two planes are called *images* of each other. Hence, by means of equations (1), a region or curve of *xy*-plane is said to *transform* or *mapped on* or represented by the corresponding curve of the *uv*-plane.

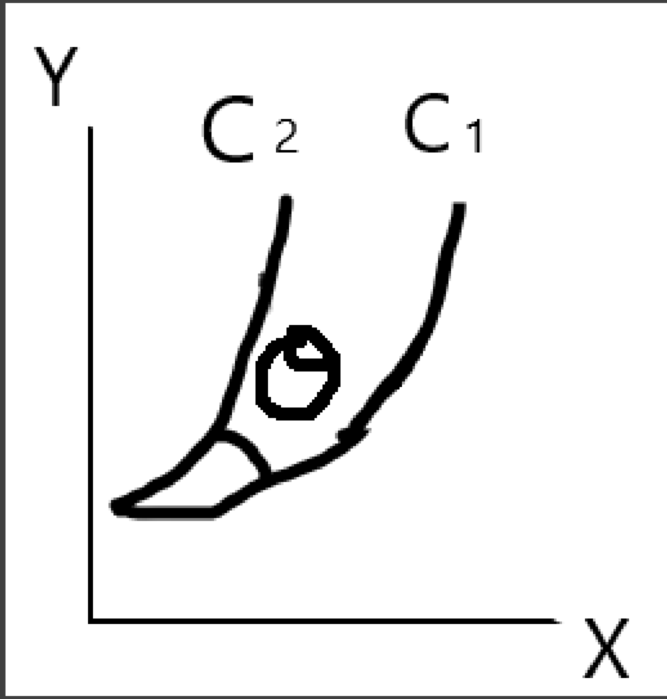
Conformal Transformation

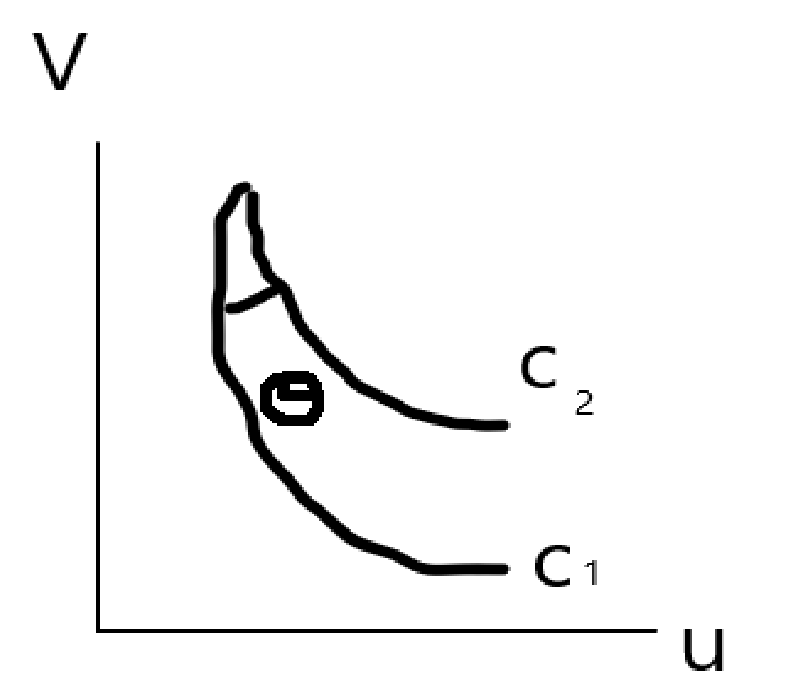
Suppose the *transformation $u = u(x,y)$, $v = (x,y)$* maps the two curves C_1, C_2 [intersecting at the point $P(z_0)$] of z-plane on the two curves C_1', C_2' [intersecting at $P'(z_0)$] of w-plane.

If the angle between C_1 and C_2 at z_0 is equal to the angle between C_1' and C_2' at w_0, then the transformation is called *isogonal*. If the sense of rotation as well as the magnitude of the angle is preserved, the transformation is called *conformal*.

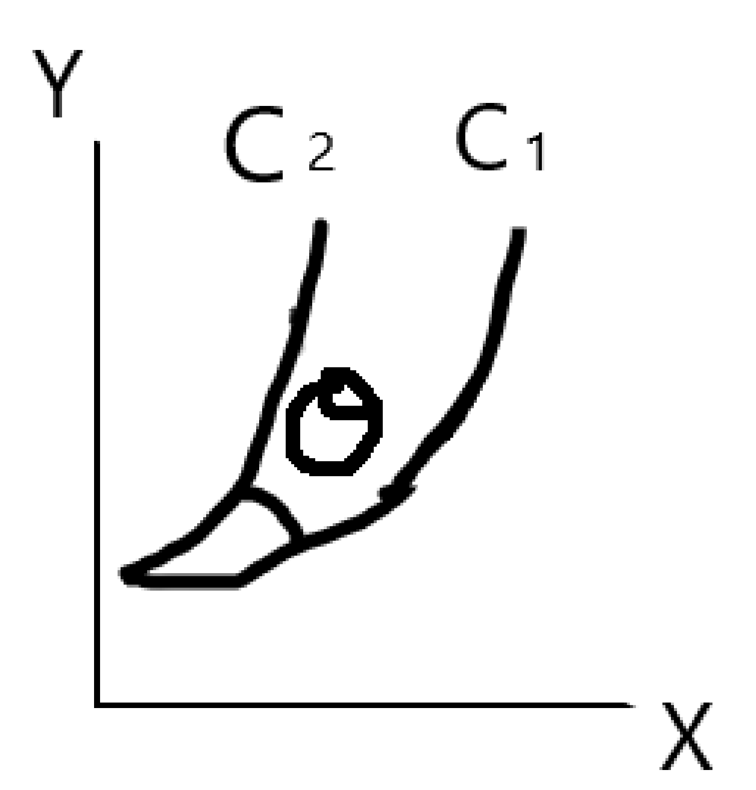
Let $w = \bar{z}$ is isogonal transformation.

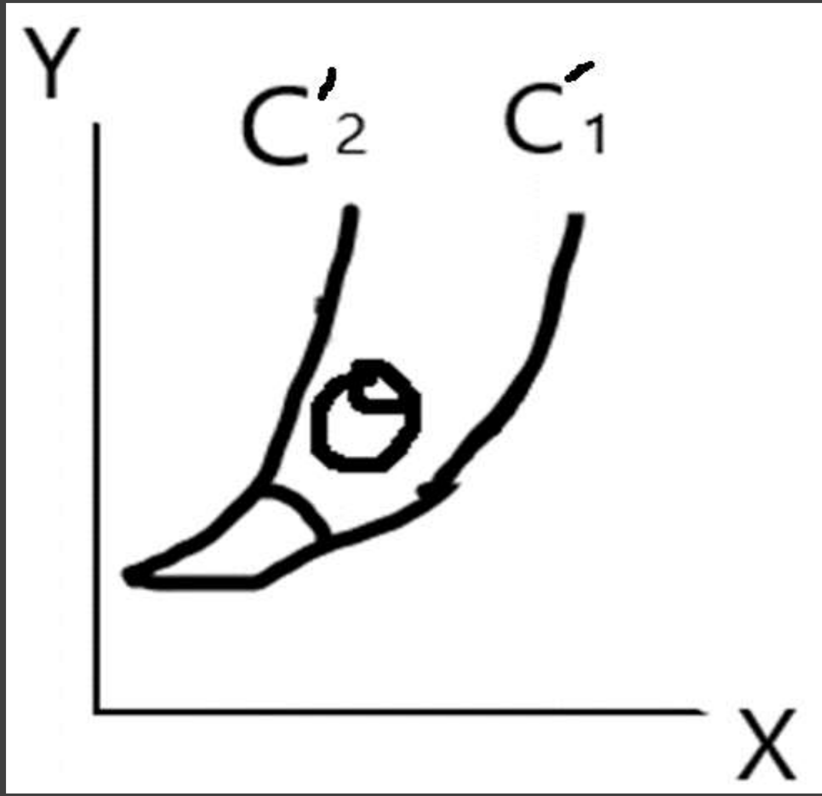
Conformal Trasformation





Isogonal Transformation





Some General Transformations

We study the following general transformations which are used in conformal transformations –

(1) Translation

(2) Rotation

(3) Magnification

(4) Inversion

Now we describe these with suitable examples.

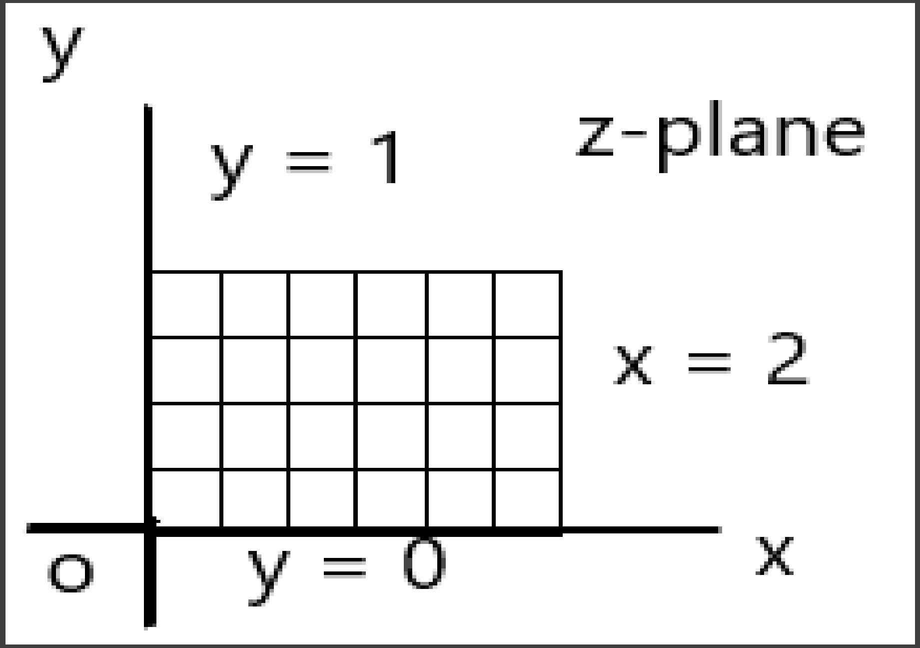
(1) Translation – The map $w = z+\beta$ corresponds to a translation. For, by this transformation, the figure in w-plane is the same as the figure in z- plane with a different origin.

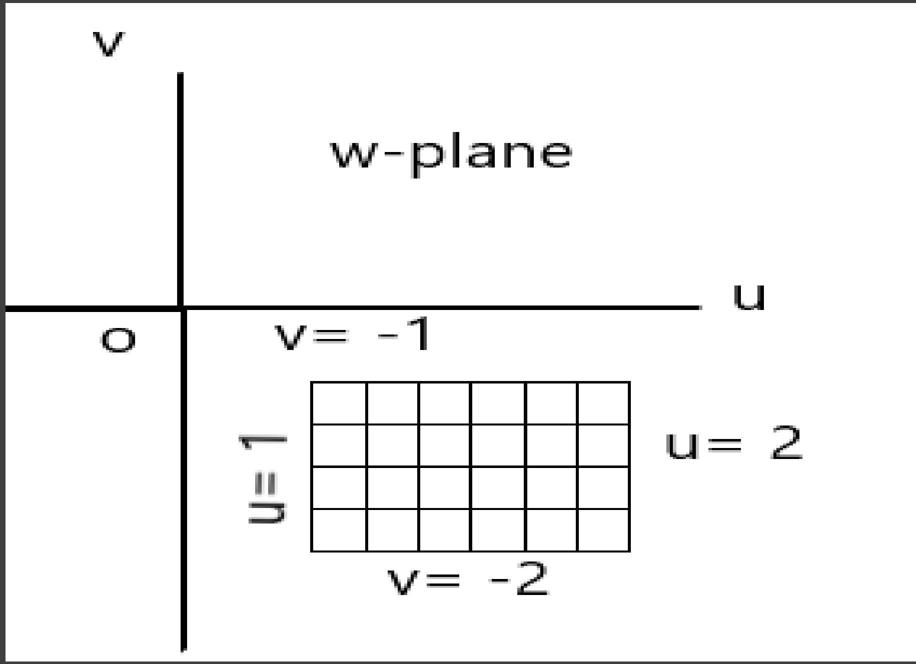
Example -

Let a rectangular domain R be bounded by x=0, y=0, x=2, y=1. Determine the region R' of w-plane into which R is mapped under the transformation

$$w = z+(1-2i)$$

This gives u=1, u=3 as x=0, x=2 and v=-2, v=-1 as y=0, y=1 respectively.
Hence the required image is rectangle R' bounded by u=1, u=3, v=-2, v=-1 in w-plane as shown in the figure.





(2) Rotation-

By the transformation w = z $e^{i\theta_0}$,

figures in z-plane are rotated through an angle θ_0. If θ_0 > 0, the rotation is anti-clockwise and θ_0 < 0, the rotation is clockwise.

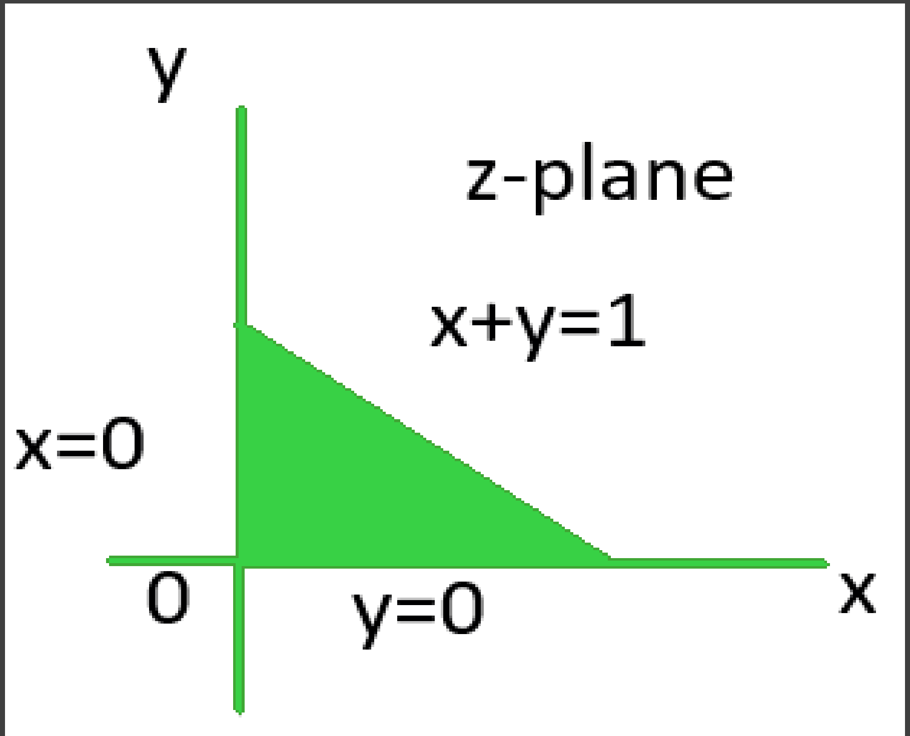
Example-

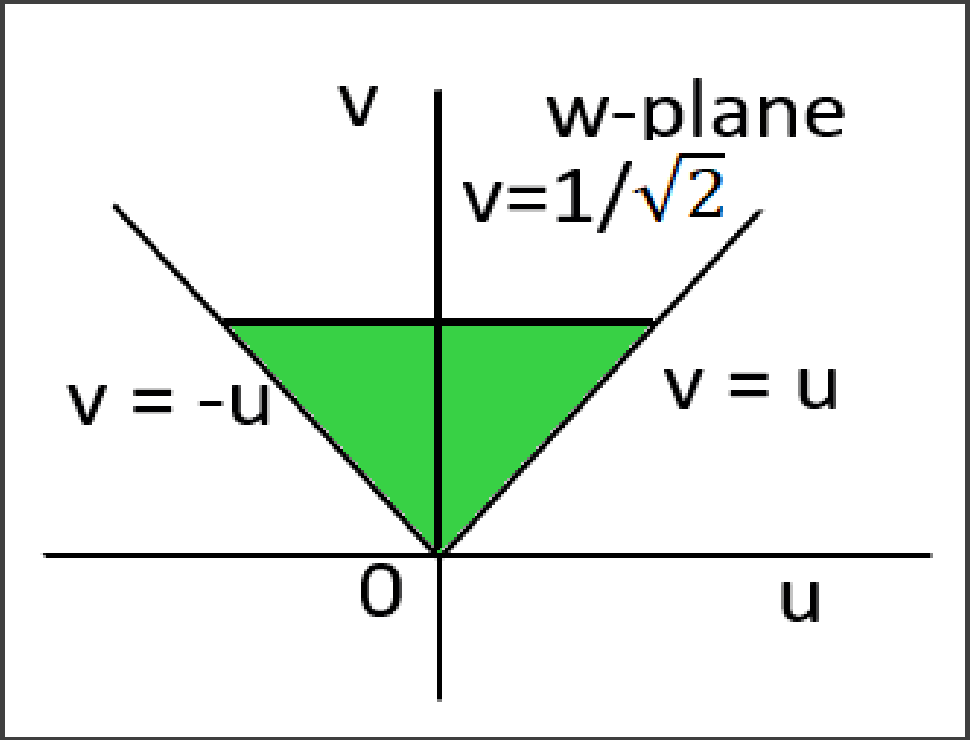
Consider the transformation w = z $e^{i\pi/4}$ and determine the region R' in w-plane corresponding to the triangular region R bounded by the lines x= 0, y= 0, and x+y= 1 in z-plane.

This gives v = -u, v = u when we put x = 0, y =0 respectively and at x+y =1, v = $1/\sqrt{2}$.

The two regions are shown in the following figure.

The mapping w = z $e^{i\pi/4}$ performs a rotation of R through an angle π/4.





(3) Magnification (Stretching)-

Consider the map $w = az$, where a is real. The two figure in z-plane and w-plane are similar and similarly situated about their respective origin, but the figure in w-plane is a times the figure in z-plane.

Such map is called magnification or stretching or dilation.

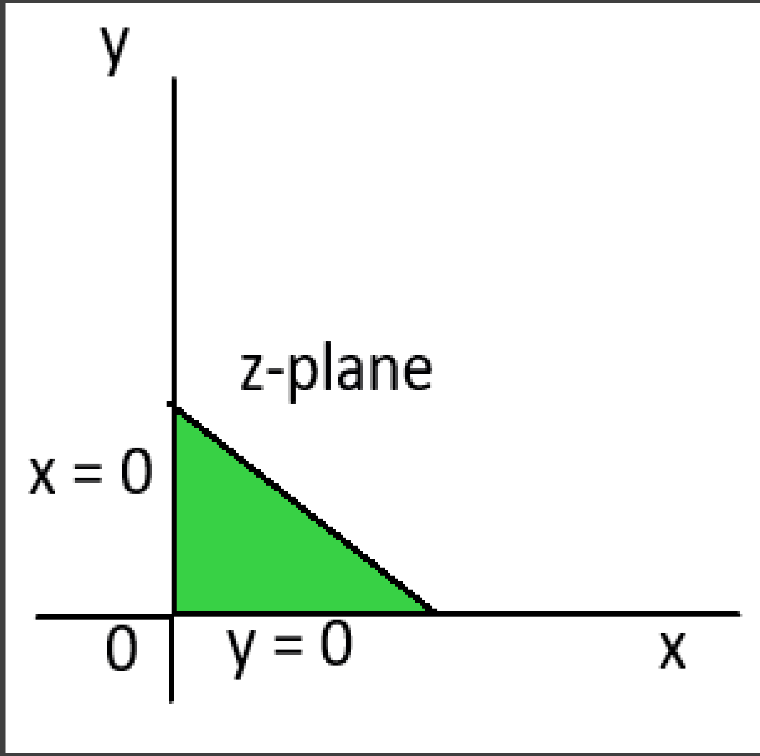
Example-

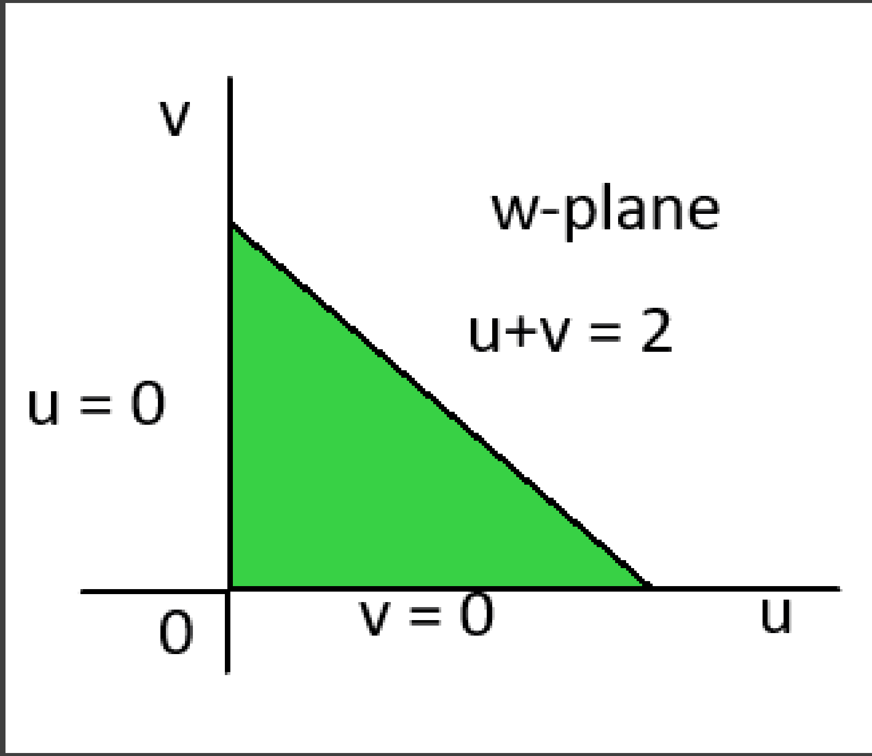
Consider the transformation $w = 2z$, and determine the region R’ of w-plane into which the triangular region R enclosed by the lines $x=0$, $y=0$, $x+y=1$ in the z-plane is mapped under the map.

Which gives us $u = 0$, $v = 0$ at $x = 0$, $y = 0$ respectively,

and $u+v = 2$ at $x+y = 1$.

This transformation w = 2z performs a magnification of R into R'.





(4) Inversion-

By means of the transformation w = 1/z, figure in z-plane are mapped upon the reciprocal figures in w-plane.

Example –

Consider the map w = 1/z and determine the region R' in w-plane of the infinite strip R bounded by 1/4 < y < 1/2.

Finally

1/4 < y < 1/2 => u^2+$(v+2)^2$ < 2^2

and u^2+$(v+1)^2$ > 1.

This shows that region R' in w-plane is bounded by the two circles.