GROUP THEORY

NO- 4

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EFFECT OF THE VARIOUS SYMMETRY OPERATIONS ON A POINT WHOSE COORINATES ARE (X,Y,Z)

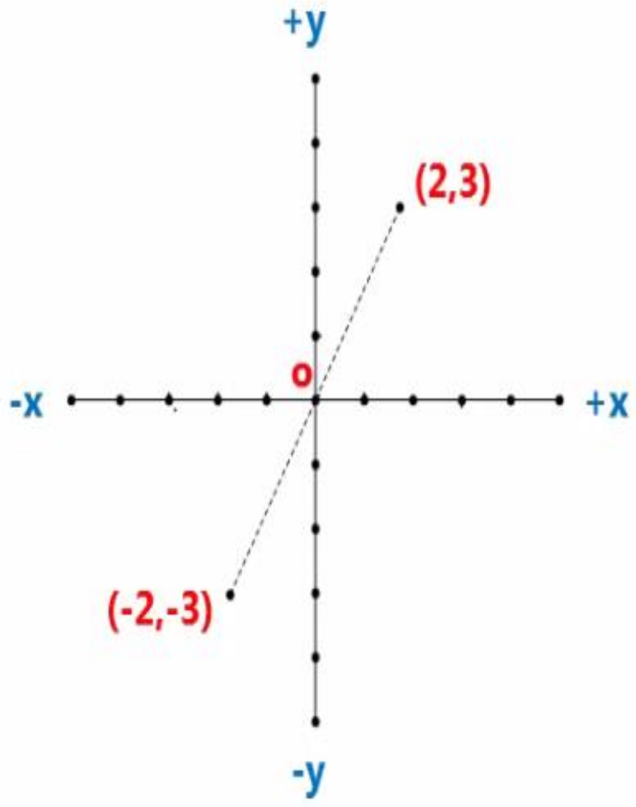
A simple but **very important concept** which will be used extensively in the study of group postulates (Next Video)

THE DIMENSIONS OF THE VARIOUS SYMMETRY ELEMENTS

SYMMETRY ELEMENT	GEOMETRY	DIMENSION
Mirror Plane σ	Plane	2
Proper Rotation Axis C_n	Line	1
Inversion Center i	Point	0
Identity E		3

Effect of symmetry operations on a point whose coordinates are (x,y,z)

When a point is subjected to a symmetry operation, there may be changes in the coordinates of the point. The number coordinates of the point that will remain **unchanged** is equal to the dimension of the symmetry element associated with the symmetry operation



The z-axis passes through the origin '0' and perpendicular to the plane of the screen

Suppose, the point (2,3) is rotated about the z-axis through 180^0 .

The point is shifted from the first quadrant to the third quadrant

and (2,3) becomes (-2,-3).That is the signs of x and y coordinates change;

x becomes –x and y becomes -y. The sign of z-coordinate of the point does

not change as this coordinate lies on the axis of rotation (z)

The dimension of proper rotation axis is 1 and hence during rotation about the z-axis only z- coordinate remain unaffected.

During rotation about the z-axis through all other angles also x and y will change and z will remain unaffected.

For example, during rotation through 90^0, the point is shifted to the second quadrant. But x and y will not become –x and –y. Only for rotation through 180^0 , x becomes –x and y become -y

How the x and y will change during rotation about z-axis through any angle other than 180^0 will be discussed in detail later

Effect of Mirror Plane σ on a point whose coordinates are (x, y, z)

- The dimension of the Mirror Plane σ is 2 . Hence 2 coordinates will remain unchanged and 1 coordinate will change sign.
- The 2 coordinates that will remain unaffected are the coordinates of the plane
- Thus, during reflection in the σ_{xy} plane x and y coordinates will remain unchanged and z coordinate will become −z
- Similarly, during reflection in the σ_{xz} plane x and z coordinates will remain unchanged and y will become −y and during reflection in the σ_{yz} plane y and z coordinates will remain unchanged and x will become −x
- These changes may b represented as

$\sigma_{xy}(x, y, z) = (x, y, -z)$ $\sigma_{xz}(x, y, z) = (x, -y, z)$ $\sigma_{yz}(x, y, z) = (-x, y, z)$

Effect of Proper Rotation axis C_2 on a point whose coordinates are (x, y, z)

- We shall confine ourselves only to rotation through 180^0 ie. the C_2 axes

  C_2^x (rotation about x-axis),

  C_2^y (rotation about y-axis) and

  C_2^z (rotation about z-axis)

- The dimension of the Proper Rotation axis C_n is 1 . Hence, 1 coordinate will remain unchanged and 2 coordinate will change sign. The coordinate which will not change its sign is the coordinate about which the rotation is effected

  C_2^x (x, y, z) = (x, -y, -z)

  C_2^y (x, y, z) = (-x, y, -z)

  C_2^z (x, y, z) = (-x, -y, z)

Effect of Inversion 'i' on a point whose coordinates are (x, y, z)

The dimension of inversion 'i' is 0 . Hence, none of the 3 coordinates will remain unchanged.

In other words all the three coordinates will change. Since inversion is actually reflection about a point, **x, y and z will become –x, -y and –z respectively**

This is denoted as

i (x, y, z) = (-x, -y, -z)

Effect of Improper rotation S_2 on a point whose coordinates are (x, y, z)

$S_2 = C_2^z \sigma_{xy}$ The order of operation is from Right to Left (Convention)

or $S_2 = C_2^z \sigma_{xy}$ means the operation σ_{xy} is performed first followed by C_2^z

$S_2 = \sigma_{xy} C_2^z$ $S_2 = \sigma_{xy} C_2^z$ means the operation C_2^z is performed first followed by σ_{xy}

$S_2 (x, y, z) = \sigma_{xy} C_2^z (x, y, z)$

$= \sigma_{xy} (-x, -y, z)$

$= (-x, -y, -z)$ **(During reflection about $\sigma_{xy,}$ -x and –y remain –x and –y and z becomes –z)**

Also i (x, y, z) = (-x, -y, -z)

Therefore S_2 = i

Effect of Identity E on a point whose coordinates are (x, y, z)

The dimension of the identity E is 3. Hence all the 3 coordinates will remain unchanged.

$$E\ (x, y, z) = (x, y, z)$$

MULTIPLICATION OF SYMMERY OPERATIONS

MULTIPLICATION OF SYMMERY OPERATIONS

- Successive operations with two or symmetry elements is called multiplication of symmetry operations

- If A and B are two different symmetry operations, AB is called the product of the two operations

- AB means the two operations A and B are performed successively

- The operation B is carried out first followed by B

- Thus the order of multiplication is from right to left

Find out the single operations corresponding to the following products

(i) $C_2^z \sigma_{xz}$ (ii) $C_2^z \sigma_{yz}$ (iii) $C_2^z C_2^z$ (iv) $\sigma_{xz} \sigma_{y`z}$ (v) $\sigma_{xz} \sigma_{xz}$ (vi) σ_{yz} σ_{yz}

(vii) E C_2^z (viii) E σ_{xz} (ix) E σ_{yz} (x) $C_2^x C_2^y$ (xi) $C_2^x C_2^z$ (xii) $C_2^y C_2^z$

(xiii) $C_2^x C_2^x$ (xiv) $C_2^y C_2^y$ (xv) $C_2^z C_2^z$

Sample:

$C_2^z \sigma_{xz}$ (x, y, z) = C_2^z (x, -y, z) = (-x, y, z)

Two coordinates y and z are not changed.

Therefore, the single operation should be 2-dimensional.

The 2-dimensional operation is reflection in a plane.

Since the coordinates which are not changed are y and z , the single operation corresponding to the product $C_2^z \sigma_{xz}$ is the mirror plane σ_{yz}

Therefore $\mathbf{C_2^z \sigma_{xz} = \sigma_{yz}}$

COMMUTING SYMMETRTY OPERATIONS

COMMUTING OPERATIONS

- If A and B are two special operations such that AB = BA (the result is the same irrespective of the order in which the operations are carried out), then A and B are said to commute. In other words, the multiplication of A and B is commutative

- Consider the two operations $\mathbf{C_2^z}$ **and** $\boldsymbol{\sigma_{xz}}$

$$C_2^z \sigma_{xz} (x, y, z) = C_2^z (x, -y, z) = (-x, y, z) = \sigma_{yz} (x, y, z)$$

Therefore $C_2^z \sigma_{xz} = \sigma_{yz}$ (1)

$$\sigma_{xz} C_2^z (x, y, z) = \sigma_{xz} (-x, -y, z) = (-x, y, z) = \sigma_{yz} (x, y, z)$$

Therefore $\sigma_{xz} C_2^z = \sigma_{yz}$ (2)

From (1) and (2)

$$C_2^z \sigma_{xz} = \sigma_{xz} C_2^z$$

That is C_2^z and σ_{xz} commute

Exercise:

Show that the following pairs of operations commute

(i) C_2^z and σ_{xz} (ii) C_2^z and σ_{yz} (iii) σ_{xz} and $\sigma_{y'z}$ (iv) E and C_2^z (v) E and σ_{xz} (vi) E and σ_{yz}

(vii) C_2^x and C_2^y (viii) C_2^x and C_2^z (xi) C_2^y and C_2^z

Continued...............